ADAPT

Technical Note
Structural Concrete Software System

TN192_Builder_punching_shear_aci_2
102505

PUNCHING SHEAR CALCULATIONS[1]
ACI – 318; ADAPT-BUILDER

1. OVERVIEW

Punching shear calculation applies to column-supported slabs, classified as two-way structural systems.

This writing (i) defines the different conditions for punching shear calculation, (ii) presents the relationships used for code check of each condition using ACI-318, (iii) presents a numerical example for each condition, and (iv) demonstrates that the program ADAPT-BUILDER correctly recognizes each case, and accordingly. This writing also serves as a guideline for verification of punching shear calculations reported by ADAPT-BUILDER.

Depending on the location of a column with respect to the slab edges, four conditions are identified. These are:

- **Interior** column, where the distance from each face of a column to the slab edge is at least four times the slab thickness (columns 4 and 5 in **Fig. 1-1**);
- **Edge** column, where one face of a column in direction of design strip is closer to the slab edge in the same direction by four times the slab thickness (column 2 in **Fig. 1-1**);
- **Corner** column, where two adjacent faces of a column are closer to their associated slab edges by less than four times the slab thickness (column 1 in **Fig. 1-1**);
- **End** column, where a column face is closer to a slab edge normal to the design strip by less than four times the slab thickness (column 6 in **Fig. 1-1**)

Columns at re-entrant corners, such as Column 3 in Fig. 1-1 are conservatively treated as edge columns. Punching shear relationships of the code do not apply to columns that are connected to one or more beams, nor do they apply to walls/supports. Adequacy of shear transfer in such cases has to be established differently.

The calculations are presented by way of a numerical example. The geometry, material, loading and other particulars of the structure selected for the numerical example are given below and in **Fig.1-1.**

Thickness of slab = 9 in (229 mm)

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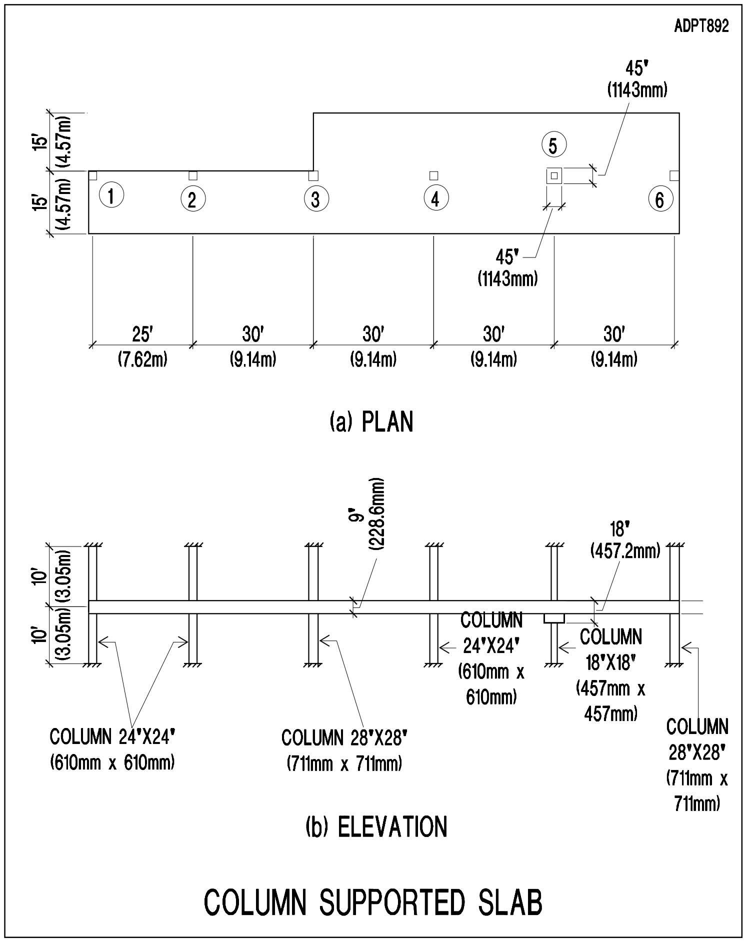


FIGURE 1-1

(i) Material Properties

- Concrete:

Compressive strength, f'_c	= 4000 ksi	(27.58 MPa)
Weight	= 150 pcf	(2403 kg/m^3)
Modulus of Elasticity	= 3605 ksi	(24856 MPa)

- Prestressing:

Low Relaxation, Unbonded System

Strand Diameter	= ½ in	(13 mm)
Strand Area	= 0.153 in^2	(98 mm^2)
Modulus of Elasticity	= 28000 ksi	(193054 MPa)
Ultimate strength of strand, f_{pu}	= 270 ksi	(1862MPa)
Minimum strand cover		
From top fiber	= 1 in all spans	(25 mm)
From bottom fiber		
Interior spans	= 1 in	(25 mm)
Exterior spans	= 1 in	(25 mm)

- Nonprestressed Reinforcement:

Yield stress f_y	= 60 ksi	(413.69 MPa)
Modulus of Elasticity	= 29000 ksi	(199,949 MPa)
Minimum Rebar Cover	= 0.75 in Top and Bottom	(19 mm)

(ii) Loading

Dead load	= self weight + 20 psf (superimposed)	
Live load	= 40 psf	(1.92 kN/m^2)

1.1. Relationships

The calculations are intended to determine whether or not a given slab-column connection meets the minimum safety requirements of the code against failure. It is not the intent of the calculations to find the "actual" condition of stress distribution at the column-slab location. The relationships used are empirical. Using test results, the relationships are calibrated to deliver safe designs.

The calculation steps are:

- Determine the factored column moment (design moment M_u) and the factored shear (design shear V_u). In many instances, column reaction is conservatively used as design value for punching shear.
- Consider a fraction of the unbalanced moment (γM_u) to contribute to the punching shear demand. The unbalanced moment is conservatively taken as the sum of upper and column moments at a joint.
- Using the code relationships, select an assumed (critical) failure surface and calculate a hypothetical maximum punching shear stress for the assumed surface.
- Using the geometry of the column-slab location and its material properties, calculate an "allowable" punching shear stress.
- If the maximum punching shear stress calculated does not exceed the allowable value, the section is considered safe.
- If the hypothetical maximum punching shear stress exceeds the allowable value by a moderate amount, punching shear reinforcement may be provided to bring the connection within the safety requirements of the code. The design of punching shear reinforcement is not covered in this writing.
- If the hypothetical maximum punching shear reinforcement exceeds the allowable values by a large margin, the section has to be enlarged.

The basic relationship is as follows:

$$v_u = \frac{V_u}{A_c} + \frac{\gamma \times M_u \times c}{J_c} \qquad (1\text{-}1)$$

Where,

V_u = absolute value of the direct shear;
M_u = Unbalanced column moment;
A_c = area of concrete of assumed critical section;

γ_v = fraction of the moment transferred by shear;
c = distance from centroidal axis of critical section to the perimeter of the critical section in the direction of analysis; and
J_c = a geometry property of critical section, analogues to polar moment of inertia of segments forming area A_c.

The first critical shear failure plane is assumed at a distance *d/2* from the face of support. Where "d" is the effective depth of the section.

Expressions for A_c, J_c, and γ_v for all types of columns are given below.

(i) Interior Column (Fig. 1.1-1)

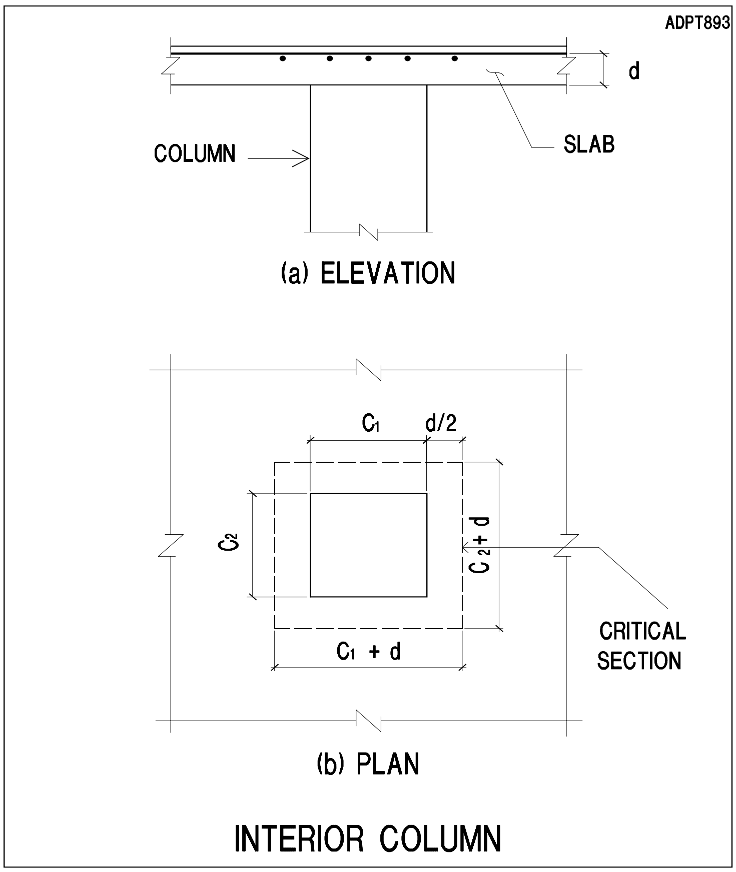


FIGURE 1.1-1

$$Ac = 2d(c_1 + c_2 + 2d)$$
$$J_c = (c_1 + d) * d^3/6 + (c_1 + d)^3 * d/6 + d * (c_2 + d) * (c_1 + d)^2 /2$$
$$\gamma_V = 1 - \{1/[1 + (2/3) * ((c_1 + d) / (c_2 + d))^{½}]\}$$

Where c_1 and c_2 are the column dimensions with c_1 perpendicular to the axis of moment, and d is the effective depth.

(ii) End Column (Refer Fig. 1.1-2)

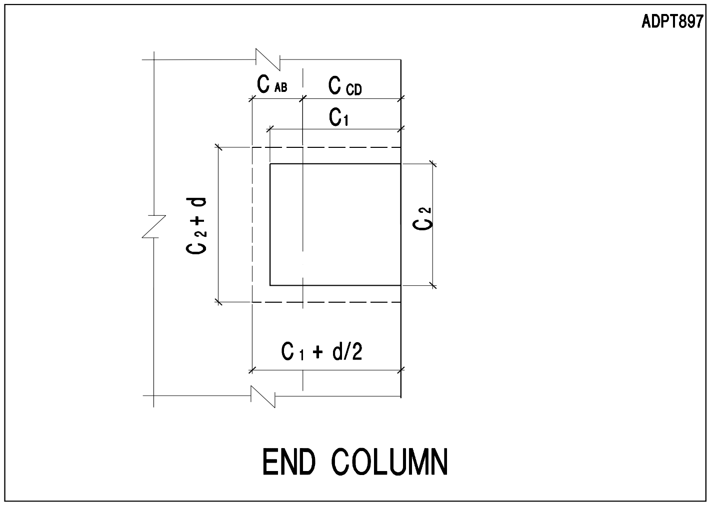


FIGURE 1.1-2

$$Ac = d\,(2c1 + c2 + 2d)$$
$$c_{AB} = (c_1 + d/2)^2 \,/\, (2c_1 + c_2 + 2d)$$
$$c_{CD} = (c_1 + d/2) - c_{AB}$$
$$J_c = (c_1 + d/2) * d^3/6 + 2d * (c_{AB}^3 + c_{CD}^3) / 3 + d * (c_2 + d)\, c_{AB}^2$$
$$\gamma_V = 1 - \{1/[1 + (2/3) * ((c_1 + d/2) / (c_2 + d))^{1/2}]\}$$

Where c_1 and c_2 are the column dimensions with c_1 parallel to the axis of moment, and d is the effective depth.

(iii) Edge Column (Refer Fig. 1.1-3)

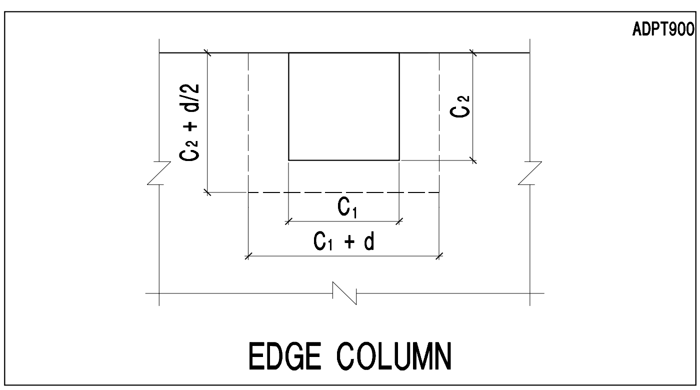


FIGURE 1.1-3

$$Ac = d\,(2c_2 + c_1 + 2d)$$
$$J_c = (c_1 + d)^3 * d/12 + (c_1 + d) * d^3/12 + d * (c_2 + d/2) * (c_1 + d)^2 /2$$
$$\gamma_V = 1 - \{1/[1 + (2/3) * ((c_1 + d) / (c_2 + d/2))^{1/2}]\}$$

Where c_1 and c_2 are the column dimensions with c_1 perpendicular to the axis of moment and d is the effective depth.

Column at the re-entrant corner as shown in **Fig.1.1- 4** is treated as Edge-column.

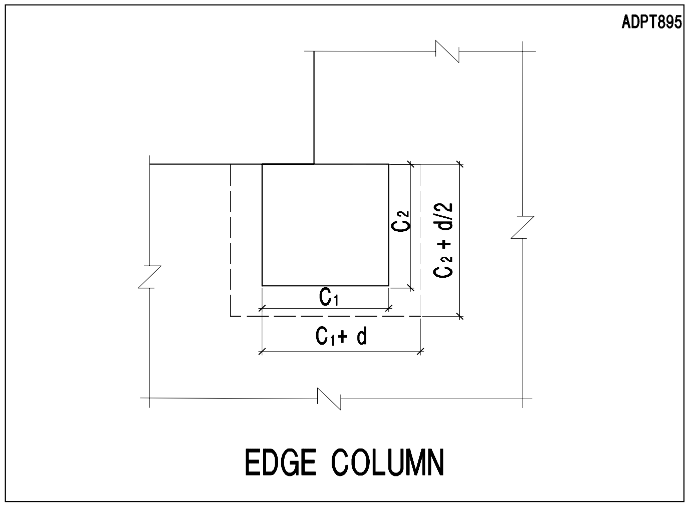


FIGURE 1.1-4

(iv) Corner Column (Refer Fig. 1.1-5)

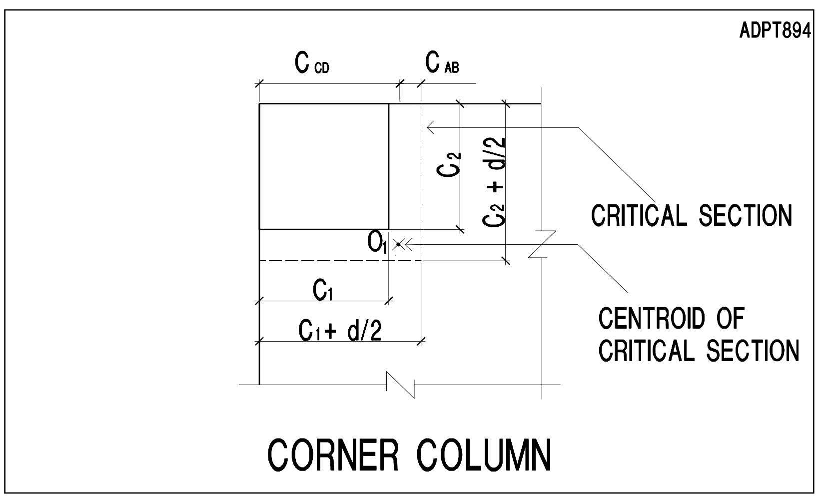


FIGURE 1.1- 5

$$Ac = d\,(c1 + c2 + d)$$
$$c_{AB} = (c_1 + d/2)^2 \;/\; 2 * (c_1 + c_2 + d)$$
$$c_{CD} = (c_1 + d/2) - c_{AB}$$
$$J_c = (c_1 + d/2) * d^3/12 + d * (c_{AB}^3 + c_{CD}^3) / 3 + d * (c_2 + d/2)\, c_{AB}^2$$
$$\gamma_V = 1- \{1/[1+ (2/3) * ((c_2 + d/2) / (c_1 + d/2))^{1/2}]\}$$

Where c_1 and c_2 are the column dimensions with c_1 parallel to the axis of moment and d is the effective depth.

For corner columns (Fig. 1.1-6) the column reaction does not act at the centroid of the critical section. The governing moment for the analysis of the design section is:

$$M_{ue} = Mu - Vu^{*}\ e$$

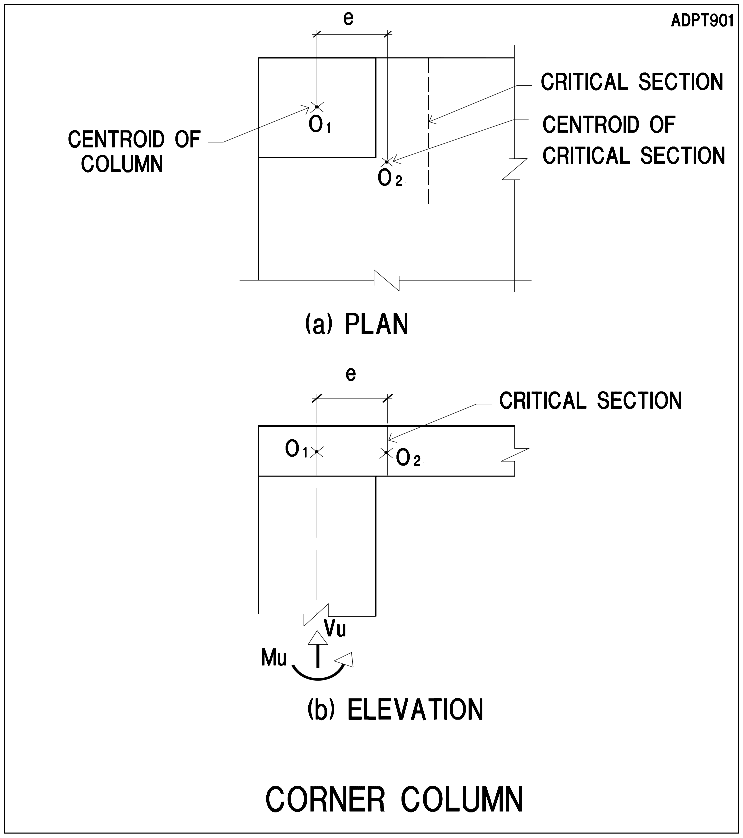


FIGURE 1.1- 6

(v) Support with Drop Cap (Refer Fig. 1.1-7)

For supports provided with drop caps, or drop panels , a minimum of two punching shear checks are necessary. The first check is at distance "$d_1/2$" from the face of the column, where d_1 is the effective depth of the thickened section (drop cap or drop panel). The second check is at a distance $d_2/2$ from the face of drop cap/panel, where d_2 is the slab thickness.

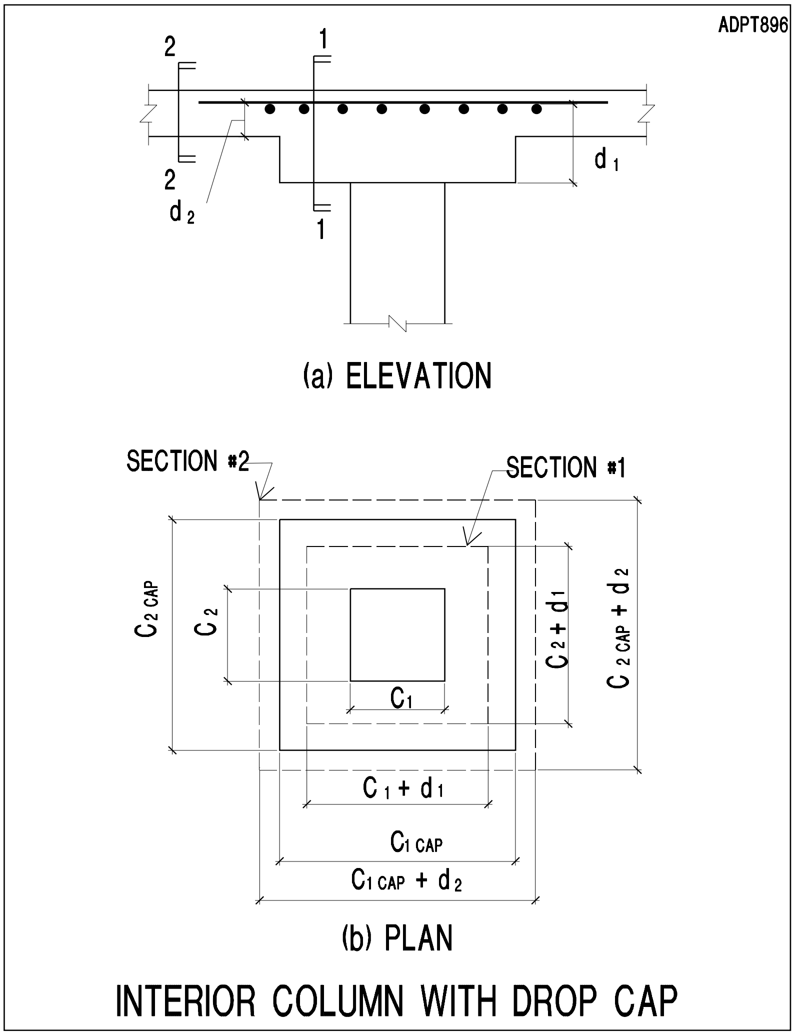


FIGURE 1.1-7

1.2. Punching Shear Stress Calculations

In order to keep the focus on punching shear stress calculation, the work starts by assuming that the design values (Mu and Vu) for each column-slab condition are given. In the general case, these are calculated from the analysis of a design strip, using the Equivalent Frame Method, or Finite Elements. The values used in this writing are obtained from an ADAPT-BUILDER computer run. The hand calculations of the stresses are compared with the computer output for verification. Excellent agreement is obtained.

A. Support #1 – corner column (Refer Fig. 1.1- 5)

Actions at the joint are:

Vu= 41.194 kips (183.24 kN)
Mu= 251.965 kip-ft (341.61 kN-m)

i. Section Properties for Shear Stress Computations

Column width,	c_1	= 24 in	(610 mm)
Column depth,	c_2	= 24 in	(610 mm)
Slab depth,	h	= 9 in	(229 mm)
Rebar used #5, diameter		= 0.625 in	(16 mm)
Top Cover to rebar		= 0.75 in	(19 mm)
	d	= 9- 0.75- 0.625 =7.625 in	(194 mm)

Since top bars in one direction are placed above the top bars in the other direction, the d value in this case is measured from the bottom of the slab to the bottom of the top layer of rebar.

For corner columns (Fig. 1.1-6) the column reaction does not act at the centroid of the critical section. The governing moment for the analysis of the design section is:

$$M_{ue} = Mu - Vu^* e$$

Where "e" is the eccentricity between the centroid of the column and that of the critical section being considered.

$c_1 + d/2 = 24 + (7.625/2) = 27.813$ in (706 mm)

$c_2 + d/2 = 24 + (7.625/2) = 27.813$ in (706 mm)

$Ac = d\,(c_1 + c_2 + d) = 7.625 * (24+ 24+ 7.625)$

$= 424.14\ in^2$ (2.736e+5 mm^2)

$c_{AB} = (c_1 + d/2)^2 / 2 * (c_1 + c_2 + d)$

$= 27.813^2 / (2* (24+ 24+ 7.625))$

$= 6.953$ in (177 mm)

$c_{CD} = (c_1 + d/2) - c_{AB}$

$= 27.813 - 6.953 = 20.860$ in (530 mm)

$J_c = (c_1 + d/2)*d^3/12+d*(c_{AB}^3 + c_{CD}^3)/3 +d* (c_2 + d/2)\, c_{AB}^2$

$= 27.813* 7.625^3/12+ 7.625 *(6.953^3 + 20.860^3)/3$

$+ 7.625 *27.813* 6.953^2$

$= 35{,}205\ in^4$ (1.465e+10 mm^4)

$\gamma_V = 1- \{1/[1+ (2/3) * ((c_2 +d/2) / (c_1 +d/2))^{½}]\}$

$= 1- \{1/[1+ (2/3) * (27.813 / 27.813)^{½}]\}$

$= 0.40$

ii. Stress Due To Direct Shear

Vu / Ac = 41.194/ 424.14

= **0.097 ksi** (0.67 MPa)

(**ADAPT-BUILDER 0.097 ksi**)

iii. Stress Due To Bending

For the first support, if the column moment is clockwise, the moment due to shear must be deducted from the column moment.

Eccentricity, $e = (c_1 + d/2) - c_{AB} - c_1/2 = 27.813 – 6.953 – 12$
$= 8.860$ in (225 mm)

$M_{ue} = 251.965 – 41.194 * 8.860 /12$
$= 221.550$ kip-ft (300.38 kN-m)

$M_{stress} = (\gamma_V * M_{ue} * c_{AB})/ J_c$
$= (0.40 * 221.55 * 12 * 6.953)/ 35,205$
$=$ **0.210 ksi** (1.45 MPa)
(**ADAPT-BUILDER 0.210 ksi**)

iv. Total Stress

Total Stress = Stress due to shear + stress due to bending
= 0.097 + 0.210
= **0.307 ksi** (2.12 MPa)
(**ADAPT-BUILDER 0.307 ksi**)

v. Allowable Stress

Column cross section is closer to a discontinuous edge than 4 times the slab thickness. Therefore, according to ACI-318-02 section 11.12.2.2, allowable stress shall be computed according to section 11.12.2.1.

∴Allowable stress is the least of

- $\phi v_c = \phi *(2 + 4/\beta_c)* \sqrt{f'_c}$
  $\phi = 0.75$
  β_c = long side of column/ short side of column
  $= 24/24 = 1$
  $\therefore \phi v_c = 0.75 *(2 + 4/1)* \sqrt{4000}/1000$
  $= 0.285$ ksi (1.97 MPa)

- $\phi v_c = \phi *((\alpha_s* d/ b_0)+ 2)* \sqrt{f'_c}$
  $\alpha_s = 20$ for corner columns
  $d = 7.625$ in (194 mm)
  b_0 = Perimeter of the critical section
  $= 2 * 27.813 = 55.626$ in (1413 mm)
  $\phi v_c = 0.75 *((20 * 7.625/ 55.626)+ 2)* \sqrt{4000} /1000$
  $= 0.225$ ksi (1.55 MPa)

- $\phi v_c = \phi *4* \sqrt{f'_c}$
  $= 0.75 * 4 * \sqrt{4000}/1000$
  $= 0.190$ ksi (1.31 MPa) ----------------- Controls

$\therefore$ Allowable Stress = **0.190 ksi** (1.31 MPa)
(**ADAPT-BUILDER 0.190 ksi**)

vi. Stress Ratio

Stress Ratio = Actual / Allowable
= 0.307/0.190
= **1.62** >1 **N.G**
(**ADAPT-BUILDER 1.62**)

For $4\sqrt{f'c}$ allowable stress, according to ACI-318-02 section 11.12.3.2, the maximum allowed is 1.5 times the permissible value. Therefore enlarge the section resisting the punching shear.

B. Support #2 – edge column (Refer Fig. 1.1-3)

Actions at the joint are:
Vu= 103.761 kips (461.55 kN)
Mu= 484.297 kip-ft (656.61 kN-m)

i. Section Properties For Shear Stress Computations

Column width,	c_1	= 24 in	(610 mm)
Column depth,	c_2	= 24 in	(610 mm)
Slab depth,	h	= 9 in	(229 mm)
Rebar used #5, diameter		= 0.625 in	(16 mm)
Top Cover to rebar		= 0.75 in	(19 mm)
	d	= 9- 0.75- 0.625 =7.625 in	(194 mm)

Since top bars in one direction are placed above the top bars in the other direction, the d value in this case is measured from the bottom of the slab to the bottom of the top layer of rebar.

$c_1 + d$ = 24 + 7.625 = 3 1.625 in (803 mm)
$c_2 + d/2$ = 24 + 7.625/2 = 27.813 in (706 mm)
Ac = $d\,(2c_2 + c_1 + 2d)$ = 7.625 * (2*24+ 24+ 2*7.625)
= 665.28 in^2 (4.292e+5 mm^2)
J_c = $(c_1+d)^3 * d/12 + (c_1 + d)*d^3/12 + d*(c_2+d/2)*(c_1+ d)^2/2$
= $31.625^3 * 7.625 / 12 + 31.625 * 7.625^3/12 + 7.625 * 27.813 * 31.625^2/2$
= 127,318 in^4 (5.299e+10 mm^4)
γ_V = $1- \{1/[1+ (2/3) * ((c_1 +d) / (c_2 +d/2))^{½}]\}$
= $1- \{1/[1+ (2/3) * (31.625 / 27.813)^{½}]\}$
= 0.416

ii. Stress Due To Direct Shear

Vu / Ac = 103.761 / 665.28
= **0.156 ksi** (1.08 MPa)
(**ADAPT-BUILDER 0.156** ksi)

iii. Stress Due To Bending

M_{stress} = $(\gamma_V * M_u * (c_1 + d))/2 * J_c$
= (0.416 * 484.297 * 12 * 31.625)/ 2*127,318
= **0.300 ksi** (0.15 MPa)
(**ADAPT-BUILDER 0.300** ksi)

iv. Total Stress

Total Stress = Stress due to shear + stress due to bending
= 0.156+0.300
= **0.456 ksi** (3.14 MPa)
(**ADAPT-BUILDER 0.456** ksi)

v. Allowable Stress

Column cross section is closer to a discontinuous edge than 4 times the slab thickness. Therefore, according to ACI-318-02 section 11.12.2.2, allowable stress shall be computed according to section 11.12.2.1.

∴ Allowable stress is the least of

- $\phi v_c = \phi *(2 + 4/\beta_c)* \sqrt{f'_c}$
  $\phi = 0.75$
  β_c = long side of column/ short side of column
  = 24/24 =1
  ∴ $\phi v_c = 0.75 *(2 + 4/1)* \sqrt{4000}/1000$
  = 0.285 ksi (1.96 MPa)

- $\phi v_c = \phi *((\alpha_s * d/ b_0) + 2)* \sqrt{f'_c}$
  α_s = 30 for edge column
  d = 7.625 in (194 mm)
  b_0 = Perimeter of the critical section
  = 2 * 27.813 +31.625 = 87.251 in (2216 mm)
  $\phi v_c = 0.75 *((30 * 7.625/ 87.251) + 2)* \sqrt{4000}/1000$
  = 0.219 ksi (1.51 MPa)

- $\phi v_c = \phi * 4 * \sqrt{f'_c}$
  = $0.75 * 4 * \sqrt{4000}/1000$
  = 0.190 ksi (1.31 MPa) ---------------- Controls

$\therefore$ Allowable Stress = **0.190 ksi** (1.31 MPa)
(**ADAPT-BUILDER 0.190** ksi)

vi. Stress Ratio

Stress Ratio = Actual / Allowable
= 0.456 / 0.190
= **2.40 >** 1 **N.G**
(**ADAPT-BUILDER 2.40**)

For $4\sqrt{f'c}$ allowable stress, according to ACI-318-02 section 11.12.3.2, the maximum allowed is 1.5 times the permissible value. Therefore enlarge the section resisting the punching shear.

C. Support # 3 – edge column (Refer Fig. 1.1-4)

Actions at the joint are:
Vu = 155.519 kips (691.78 kN)
Mu = 197.858 kip-ft (268.26 kN-m)

i. Section Properties For Shear Stress Computations

Column width, c_1 = 28 in (711 mm)
Column depth, c_2 = 28 in (711 mm)
Slab depth, h = 9 in (229 mm)
Rebar used #5, diameter = 0.625 in (16 mm)
Top Cover to rebar = 0.75 in (19 mm)
d = 9- 0.75- 0.625 =7.625 in (194 mm)

Since top bars in one direction are placed above the top bars in the other direction, the d value in this case is measured from the bottom of the slab to the bottom of the top layer of rebar.

$c_1 + d$ = 28 + 7.625 = 35.625 in (905 mm)
$c_2 + d/2$ = 28 + 7.625/2 = 31.813 in (808 mm)
$A_c = d(2c_2 + c_1 + 2d) = 7.625 * (2*28 + 28 + 2*7.625)$
$= 756.78\ in^2$ ($4.882e+5\ mm^2$)
$J_c = (c_1+d)^3 * d/12 + (c_1 + d)*d^3/12 + d*(c_2+d/2)*(c_1+d)^2/2$
$= 35.625^3 * 7.625 /12 + 35.625 * 7.625^3/12 + 7.625 * 31.813 * 35.625^2/2$
$= 183{,}976\ in^4$ ($7.658e+10\ mm^4$)
$\gamma_V = 1 - \{1/[1 + (2/3) * ((c_1 + d) / (c_2 + d/2))^{½}]\}$
$= 1 - \{1/[1 + (2/3) * (35.625 / 31.813)^{½}]\}$
$= 0.414$

ii. Stress Due To Direct Shear

Vu / Ac = 155.519 / 756.78
= **0.206 ksi** (1.42 MPa)
(**ADAPT-BUILDER 0.205** ksi)

iii. Stress Due To Bending

M_{stress} = $(\gamma_V * M_u * (c_1 + d))/2 * J_c$
= (0.414 * 197.858 * 12 * 35.625)/ 2*183,976
= **0.095 ksi** (0.66 MPa)
(**ADAPT-BUILDER 0.095** ksi)

iv. Total Stress

Total Stress = Stress due to shear + stress due to bending
= 0.206+ 0.095
= **0.301 ksi** (2.08 MPa)
(**ADAPT-BUILDER 0.301** ksi)

v. Allowable Stress

Column cross section is closer to a discontinuous edge than 4 times the slab thickness. Therefore, according to ACI-318-02 section 11.12.2.2, allowable stress shall be computed according to section 11.12.2.1.

∴ Allowable stress is the least of

- $\phi v_c = \phi * (2 + 4/\beta_c) * \sqrt{f'_c}$
  $\phi = 0.75$
  β_c = long side of column/ short side of column
  = 28/28 = 1
  ∴ $\phi v_c = 0.75 * (2 + 4/1) * \sqrt{4000}/1000$
  = 0.285 ksi (1.96 MPa)

- $\phi v_c = \phi * ((\alpha_s * d/ b_0) + 2) * \sqrt{f'_c}$
  α_s = 30 for edge column
  d = 7.625 in (194 mm)
  b_0 = Perimeter of the critical section
  = 2 * 31.813 +35.625
  = 99.251 in (2521 mm)
  $\phi v_c = 0.75 * ((30 * 7.625/ 99.251) + 2) * \sqrt{4000}/1000$
  = 0.204 ksi (1.41 MPa)

- $\phi v_c = \phi * 4 * \sqrt{f'_c}$
  = $0.75 * 4 * \sqrt{4000}/1000$
  = 0.190 ksi (1.31 MPa) -------------- Controls

$\therefore$ Allowable Stress = **0.190 ksi** (1.31 MPa)
(**ADAPT-BUILDER 0.190** ksi)

vi. Stress Ratio

Stress Ratio = Actual / Allowable
= 0.301/ 0.190
= **1.58** > 1 **N.G**
(**ADAPT-BUILDER 1.58**)

For $4\sqrt{f'_c}$ allowable stress, according to ACI-318-02 section 11.12.3.2, the maximum allowed is 1.5 times the permissible value. Therefore enlarge the section resisting the punching shear.

D. Support #4 – interior column (Refer Fig.1.1-1)

Actions at the joint are:
Vu = 203.511 kips (691.78 kN)
Mu = 76.264 kip-ft (103.40 kN-m)

i. Section Properties For Shear Stress Computations

Column width, c_1 = 24 in (610 mm)
Column depth, c_2 = 24 in (610 mm)
Slab depth, h = 9 in (229 mm)
Rebar used #5, diameter = 0.625 in (16 mm)
Top Cover to rebar = 0.75 in (19 mm)
d = 9- 0.75- 0.625
= 7.625 in (194 mm)

Since top bars in one direction are placed above the top bars in the other direction, the d value in this case is measured from the bottom of the slab to the bottom of the top layer of rebar.

$c_1 + d$ = 24 + 7.625 = 31.625 in (803 mm)
$c_2 + d$ = 24 + 7.625 = 31.625 in (803 mm)
Ac = $2d(c_1 + c_2 + 2d)) = 2*7.625 * (24+ 24+ 2*7.625)$
= 964.56 in^2 (6.223e+5 mm^2)
J_c = $(c_1+ d)*d^3/6+ (c_1 + d)^3*d/6 +d* (c_2 + d)*(c_1+ d)^2 /2$
= $31.625*7.625^3/6 +31.625^3*7.625 /6+7.625 *31.625* 31.625^2 /2$
= 163,120 in^4 (6.790e+10 mm^4)
γ_V = $1- \{1/[1+ (2/3) * ((c_1 +d) / (c_2 +d))^{1/2}]\}$
= $1- \{1/[1+ (2/3) * (31.625 / 31.625)^{1/2}]\}$
= **0.40**

ii. Stress Due To Direct Shear

Vu / Ac = 203.514 / 964.56

$= \mathbf{0.211\ ksi}$ (1.45 MPa)
(**ADAPT-BUILDER 0.211** ksi)

iii. Stress Due To Bending

$M_{stress} = (\gamma_V * M_u * (c_1 + d))/(2 * J_c)$
$= (0.40 * 76.264 * 12 * 31.625)/ 2*163{,}120$
$= \mathbf{0.035\ ksi}$ (0.09 MPa)
(**ADAPT-BUILDER 0.035** ksi)

iv. Total Stress

Total Stress = Stress due to shear + stress due to bending
= 0.211 + 0.035
= **0.246 ksi** (1.70 MPa)
(**ADAPT-BUILDER 0.246** ksi, B12, C7)

v. Allowable Stress

From ACI-318-02 equation 11.36

Allowable Stress,

$\phi\, v_c = \phi *[(\beta_p * \sqrt{f'_c} + 0.3 * f_{pc}) + V_p]$

Where,

$\phi = 0.75$
β_p is the smaller of 3.5 or $((\alpha_s * d / b_0) + 1.5)$
$\alpha_s = 40$ for interior column
b_0 = Perimeter of the critical section
$= 4 * 31.625$
$= 126.50$ in (3213 mm)
$d = 7.625$ in (194 mm)
$\beta_p = ((\alpha_s * d / b_0) + 1.5) = ((40 * 7.625 / 126.50) + 1.5)$
$= 3.91 > 3.50, \quad \therefore$ use 3.50
$f_{pc} = P/A = 125$ ksi (0.86 MPa)

Conservatively 125 psi is used since this is a minimum code requirement.

$\phi\, v_c = 0.75 * (3.5 * \sqrt{4000} + 0.3 * 125)$
$= 0.194$ ksi (1.34 MPa)

$\therefore$ Allowable Stress = **0.194 ksi** (1.34 MPa)
(**ADAPT-BUILDER 0.194** ksi)

Note that in the evaluation of allowable stresses, the term corresponding to the vertical component of tendon force (Vp) is conservatively disregarded.

vi. Stress Ratio

Stress Ratio = Actual / Allowable
= 0.246 / 0.194
= **1.27** > 1 **N.G**
(**ADAPT-BUILDER 1.27**)

Punching Shear Stress exceeds the permissible value. Provide shear reinforcement.

E. Support #5 – interior column with drop cap (Refer Fig.1.1- 7)

Actions at the joint are:
Vu = 232.588 kips (1034.60 kN)
Mu = 149.179 kip-ft (202.26 kN-m)

Check whether the critical section lies within the cap or slab.

Section #1 (d/2 from the column face)

i. Section Properties For Shear Stress Computations

Column width,	c_1 = 18 in	(457 mm)
Column depth,	c_2 = 18 in	(457 mm)
Slab depth,	h = 9 +9 = 18 in	(457 mm)
Rebar used #5, diameter	= 0.625 in	(16 mm)
Top Cover to rebar	= 0.75 in	(19 mm)
	d_1 = 18- 0.75- 0.625 = 16.625 in	(422 mm)

Since top bars in one direction are placed above the top bars in the other direction, the d_1 value in this case is measured from the bottom of the drop panel to the bottom of the top layer of rebar.

$c_1 + d_1$ = 18 + 16.625 = 34.625 in (880 mm)
$c_2 + d_1$ = 18 + 16.625 = 34.625 in (880 mm)
Ac = $2d(c_1 + c_2 + 2d)$)= 2*16.625 * (18+ 18+ 2*16.625)
= 2302.56 in^2 (1.486e+6 mm^2)
J_c = $(c_1 + d)*d^3/6 + (c_1 + d)^3*d/6 + d*(c_2 + d)*(c_1 + d)^2/2$
= $34.625*16.625^3/6 + 34.625^3*16.625/6 + 16.625*34.625*34.625^2/2$
= 486,604 in^4 (2.025e+11 mm^4)
γ_V = $1- \{1/[1+ (2/3) * ((c_1 + d) / (c_2 + d))^{½}]\}$
= $1- \{1/[1+ (2/3) * (34.625 / 34.625)^{½}]\}$
= **0.40**

ii. Stress Due To Direct Shear

Vu / Ac = 232.588 / 2302.56
= **0.101 ksi** (0.70 MPa)

iii. Stress Due To Bending

M_{stress} = $(\gamma_V * M_u * (c_1 + d)) / (2 * J_c)$
= (0.40 * 149.179 * 12 * 34.625)/ 2*486,604
= **0.025 ksi** (0.17 MPa)

iv. Total Stress

Total Stress = Stress due to shear + stress due to bending
= 0.101 + 0.025
= **0.126 ksi** (0.87 MPa)

v. Allowable Stress

From ACI-318-02 (equation 11.36)

Allowable Stress,
$\phi\, v_c = \phi * [(\beta_p * \sqrt{f'_c} + 0.3 * f_{pc}) + V_p]$

Where,
ϕ = 0.75
β_p is the smaller of 3.5 or $((\alpha_s * d / b_0) + 1.5)$
α_s = 40 for interior column
b_0 = Perimeter of the critical section
= 4 * 34.625
= 138.50 in (3518 mm)
d = 16.625 in (422 mm)
β_p = $((\alpha_s * d / b_0) + 1.5)$ = ((40* 16.625 / 138.50)+ 1.5)
= 6.30 >3.50, $\therefore$ use 3.50
f_{pc} = P/A = 125 ksi (0.86 MPa)

Conservatively 125 psi is used since this is a minimum code requirement.

$\phi\, v_c$ = $0.75 * (3.5 * \sqrt{4000} + 0.3 * 125)$
= 0.194 ksi (1.34 MPa)

$\therefore$ Allowable Stress = **0.194 ksi** (1.34 MPa)

Note that in the evaluation of allowable stresses, the term corresponding to the vertical component of tendon force (Vp) is conservatively disregarded.

vi. Stress Ratio

Stress Ratio = Actual / Allowable
= 0.126 / 0.194
= **0.65**

Section #2 (d/2 from the drop cap face)

i. Section Properties For Shear Stress Computations

Cap width,	c_1 = 45 in		(1143 mm)
Cap depth,	c_2 = 45 in		(1143 mm)
Slab depth,	h = 9 in		(229 mm)
Rebar used #5, diameter	= 0.625 in		(16 mm)
Top Cover to rebar	= 0.75 in		(19 mm)
	d_2 = 9- 0.75- 0.625 = 7.625 in		(194 mm)

Since top bars in one direction are placed above the top bars in the other direction, the d_2 value in this case is measured from the bottom of the slab to the bottom of the top layer of rebar.

$c_{1\,CAP} + d_2$ = 45 + 7.625 = 52.625 in (1337 mm)
$c_{2\,CAP} + d_2$ = 45 + 7.625 = 52.625 in (1337 mm)
Ac = $2d(c_1 + c_2 + 2d)$ = 2*7.625 * (45+ 45+ 2*7.625)
= 1605.06 in^2 (1.036e+6 mm^2)
J_c = $(c_1+ d)*d^3/6+ (c_1 + d)^3*d/6 +d* (c_2 + d)*(c_1+ d)^2 /2$
= $(52.625 *7.625^3)/6 +(52.625^3*7.625)/6+(7.625 *52.625 * 52.625^2)/2$
= 744,729 in^4 (3.100e+11 mm^4)
γ_V = $1- \{1/[1+ (2/3) * ((c_1 +d) / (c_2 +d))^{½}]\}$
= $1- \{1/[1+ (2/3) * (52.625 / 52.625)^{½}]\}$
= **0.40**

ii. Stress Due To Direct Shear

Vu / Ac = 232.588 / 1605.06
= **0.145 ksi** (1.00 MPa)
(**ADAPT-BUILDER 0.145** ksi)

iii. Stress Due To Bending

M_{stress} = $(\gamma_V * M_u * (c_1+ d))/ (2* J_c)$
= (0.40 * 149.179 * 12 * 52.625)/ 2*744,729
= **0.025 ksi** (0.17 MPa)
(**ADAPT-BUILDER 0.025** ksi)

iv. Total Stress

Total Stress = Stress due to shear + stress due to bending
= 0.145 + 0.025

= **0.170 ksi** (1.17 MPa)

(**ADAPT-BUILDER 0.170** ksi)

v. Allowable Stress

From ACI-318-02 equation 11.36

Allowable Stress,

$\phi\, v_c = \phi * [(\beta_p * \sqrt{f'_c} + 0.3 * f_{pc}) + V_p]$

Where,

ϕ = 0.75

β_p is the smaller of 3.5 or (α_s* d/ b_0)+ 1.5)

α_s = 40 for interior column

b_0 = Perimeter of the critical section

= 4 * 52.625

= 210.50 in (5347 mm)

d = 7.625 in (194 mm)

β_p = (α_s* d/ b_0)+ 1.5) = (40* 7.625 / 210.50)+ 1.5)

= 2.95 < 3.50, $\therefore$ use 2.95

f_{pc} = P/A = 125 ksi (0.86 MPa)

$\phi\, v_c$ = 0.75 *(2.95* $\sqrt{4000}$ + 0.3 *125) /1000

= 0.168 ksi (1.16 MPa)

$\therefore$ Allowable Stress = **0.168 ksi** (1.16 MPa)

(**ADAPT-BUILDER 0.168** ksi)

Note that in the evaluation of allowable stresses, the term corresponding to the vertical component of tendon force (Vp) is conservatively disregarded.

vi. Stress Ratio

Stress Ratio = Actual / Allowable

= 0.170 / 0.168

= **1.01** > 1 **N.G**

(**ADAPT-BUILDER 1.01**)

Since the stress ratio in section#2 is larger than the stress ratio in section #1, the section#2 governs and reported in the program.

Punching Shear Stress exceeds the permissible value. Provide shear reinforcement.

F. Support #6 – end column (Refer Fig. 1.1-2)

Actions at the joint are:

Vu = 94.629 kips (420.93 kN)

Mu = 93.862 kip-ft (127.26 kN-m)

i. Section Properties For Shear Stress Computations

Column width,	c_1	= 28 in	(711 mm)
Column depth,	c_2	= 28 in	(711 mm)
Slab depth,	h	= 9 in	(229 mm)
Rebar used #5, diameter		= 0.625 in	(16 mm)
Top Cover to rebar		= 0.75 in	(19 mm)
	d	= 9- 0.75- 0.625 = 7.625 in	(194 mm)

Since top bars in one direction are placed above the top bars in the other direction, the d value in this case is measured from the bottom of the slab to the bottom of the top layer of rebar.

$c_1 + d/2 = 28 + 7.625/2 = 31.813$ in (808 mm)
$c_2 + d = 28 + 7.625 = 35.625$ in (905 mm)
$Ac = d\,(2c1 + c2 + 2d) = 7.625 * (2*28 + 28 + 2*7.625)$
$= 756.78\ in^2$ ($4.882e+5\ mm^2$)
$c_{AB} = (c_1 + d/2)^2 /(2c_1 + c_2 + 2d)$
$= 31.813^2 / (2*28 + 28 + 2*7.625)$
$= 10.200$ in (259 mm)
$c_{CD} = 31.813 - 10.200$
$= 21.613$ in (549 mm)
$J_c = 31.813 *7.625^3/6 + 2*7.625 *(10.200^3 + 21.613^3)/3$
$+7.625 *35.625*10.200^2$
$= 87{,}327\ in^4$ ($3.635e+10\ mm^4$)
$\gamma_V = 1- \{1/[1+ (2/3) * ((c_1 +d/2) / (c_2 +d))^{½}]\}$
$= 1- \{1/[1+ (2/3) * (31.813 / 35.625)^{½}]\}$
$=$ **0.386**

ii. Stress Due To Direct Shear

$Vu / Ac = 94.629 / 756.78$
$=$ **0.125 ksi** (0.86 MPa)
(**ADAPT-BUILDER 0.125** ksi)

iii. Stress Due To Bending

$M_{ue} = Mu – Vu^* e$

For the last support, if the column moment is anticlockwise, the moment due to shear must be deducted.

Eccentricity, $e = (c_1+ d/2) - c_{AB} - c_1/2 = 31.813 – 10.200 – 14$
$= 7.613$ in (193 mm)
$M_{ue} = 93.862- 94.629 * 7.613 /12$
$= 33.828$ kip-ft (45.86 kN-m)
$M_{stress} = (\gamma_V * M_{ue} * c_{AB})/ J_c$
$= (0.386 * 33.828 * 12 * 10.200)/ 87{,}327$

= **0.018 ksi** (0.12 MPa)
(**ADAPT-BUILDER 0.018 ksi**)

iv. Total Stress

Total Stress = Stress due to shear + stress due to bending
= 0.125 + 0.018
= **0.143 ksi** (0.99 MPa)
(**ADAPT-BUILDER 0.143** ksi)

v. Allowable Stress

Column cross section is closer to a discontinuous edge than 4 times the slab thickness. Therefore, according to ACI-318-02 section 11.12.2.2, allowable stress shall be computed according to section 11.12.2.1.

$\therefore$ Allowable stress is the least of

- $\phi\, v_c = \phi * (2 + 4/\beta_c) * \sqrt{f'_c}$
  $\phi = 0.75$
  β_c = long side of column/ short side of column
  = 28/28 =1
  $\therefore\ \phi\, v_c = 0.75 * (2 + 4/1) * \sqrt{4000}/1000$
  = 0.285 ksi (1.96 MPa)

- $\phi\, v_c = \phi * ((\alpha_s * d/ b_0) + 2) * \sqrt{f'_c}$
  α_s = 30 for end column
  d = 7.625 in (194 mm)
  b_0 = Perimeter of the critical section
  = 2 * 31.813 +35.625
  = 99.251 in (2521 mm)

  $\phi\, v_c = 0.75 * ((30 * 7.625/ 99.251) + 2) * \sqrt{4000}/1000$
  = 0.204 ksi (1.41 MPa)

- $\phi\, v_c = \phi * 4 * \sqrt{f'_c}$
  $= 0.75 * 4 * \sqrt{4000}/1000$
  = 0.190 ksi (1.31 MPa) ----------------- Controls

$\therefore$ Allowable Stress = **0.190 ksi** (1.31 MPa)
(**ADAPT-BUILDER 0.190** ksi)

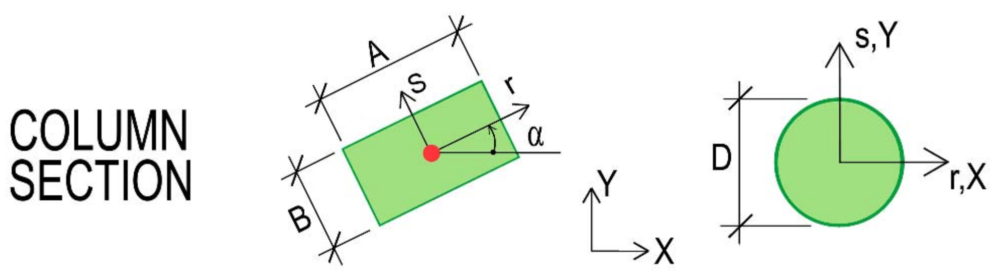
vi. Stress Ratio

Stress Ratio = Actual / Allowable
= 0.143 / 0.190
= **0.75** < 1 **OK**

(**ADAPT-BUILDER 0.76**)

The following are excerpt from the report generated by the program Floor-Pro for the punching shear calculations used in this writing.

110.50 COLUMNS



Column Dimensions and Material Property

ID	Label	Centroid X,Y,Alpha	A or D	B	Height	Material	Position
		ft ft °	in	in	ft		
1	Column 1	1.00,14.00,0.00	24.00	24.00	10.00	Concrete 1	Lower
2	Column 2	26.00,14.00,0.00	24.00	24.00	10.00	Concrete 1	Lower
3	Column 3	56.00,14.00,0.00	28.00	28.00	10.00	Concrete 1	Lower
4	Column 4	86.00,14.00,0.00	24.00	24.00	10.00	Concrete 1	Lower
5	Column 5	116.00,14.00,0.00	18.00	18.00	10.00	Concrete 1	Lower
6	Column 6	146.00,14.00,0.00	28.00	28.00	10.00	Concrete 1	Lower
7	Column 7	1.00,14.00,0.00	24.00	24.00	10.00	Concrete 1	Upper
8	Column 8	26.00,14.00,0.00	24.00	24.00	10.00	Concrete 1	Upper
9	Column 9	56.00,14.00,0.00	28.00	28.00	10.00	Concrete 1	Upper
10	Column 10	86.00,14.00,0.00	24.00	24.00	10.00	Concrete 1	Upper
11	Column 11	116.00,14.00,0.00	18.00	18.00	10.00	Concrete 1	Upper
12	Column 12	146.00,14.00,0.00	28.00	28.00	10.00	Concrete 1	Upper

180.60 PUNCHING SHEAR STRESS CHECK PARAMETERS

Label	Condition	Axis	Effective depth	Design length rr	Design length ss	Design area	Section constant	Gamma
			in	in	in	in2	in4	
Column 1	Corner	rr	7.63	27.81	27.81	4.24E+002	3.52E+004	0.40
Column 1	Corner	ss	7.63	27.81	27.81	4.24E+002	3.52E+004	0.40
Column 2	End	rr	7.63	27.81	31.63	6.65E+002	5.91E+004	0.38
Column 2	Edge	ss	7.63	31.63	27.81	6.65E+002	1.27E+005	0.42
Column 3	End	rr	7.63	31.81	35.63	7.57E+002	8.73E+004	0.39
Column 3	Edge	ss	7.63	35.63	31.81	7.57E+002	1.84E+005	0.41
Column 4	Interior	rr	7.63	31.63	31.63	9.65E+002	1.63E+005	0.40
Column 4	Interior	ss	7.63	31.63	31.63	9.65E+002	1.63E+005	0.40
Column 5	Interior	rr	7.63	52.63	52.63	1.61E+003	7.45E+005	0.40
Column 5	Interior	ss	7.63	52.63	52.63	1.61E+003	7.45E+005	0.40
Column 6	Edge	rr	7.63	35.63	31.81	7.57E+002	1.84E+005	0.41
Column 6	End	ss	7.63	31.81	35.63	7.57E+002	8.73E+004	0.39

180.40 PUNCHING SHEAR STRESS CHECK RESULTS

Load Combination: Strength(Dead and Live)

Label	Condition	Axis	Factored shear	Factored moment	Stress due to shear	Stress due to moment	Total stress	Allowable stress	Stress ratio	Case
			k	k-ft	ksi	ksi	ksi	ksi		
Column 1	Corner	rr	-41.194	251.965	0.097	0.210	0.307	0.190	1.62	1
Column 1	Corner	ss	-41.194	12.836	0.097	0.017	0.114	0.190	0.60	1
Column 2	Edge	rr	-103.761	484.297	0.156	0.300	0.456	0.190	2.40	1
Column 2	End	ss	-103.761	-0.536	0.156	0.041	0.197	0.190	1.04	1
Column 3	Edge	rr	-155.519	197.858	0.205	0.095	0.301	0.190	1.58	1
Column 3	End	ss	-155.519	197.769	0.205	0.161	0.366	0.190	1.93	1
Column 4	Interior	rr	-203.514	-76.264	0.211	0.035	0.246	0.194	1.27	1
Column 4	Interior	ss	-203.514	-49.468	0.211	0.023	0.234	0.194	1.21	1
Column 5	Interior	rr	-232.588	-149.179	0.145	0.025	0.170	0.168	1.01	2
Column 5	Interior	ss	-232.588	47.776	0.145	0.008	0.153	0.168	0.91	2
Column 6	End	rr	-94.629	-93.862	0.125	0.018	0.143	0.190	0.76	1
Column 6	Edge	ss	-94.629	-106.843	0.125	0.051	0.176	0.190	0.93	1

Legend:
CASE............1=Stress within section #1 governs (column cap or slab) and
CASE............2=Stress within section #2 governs (drop panel or slab)

CONDITION.....(a)=Program does not check for this column. No result!